

## References

- <sup>1</sup> Bogdonoff, S. M., Kepler, C. E., and Sanlorenzo, E., "A study of shock wave turbulent boundary layer interaction at  $M = 3$ ," Princeton Univ., Dept. of Aeronautical Engineering, Rept. 222 (1953).
- <sup>2</sup> Chapman, D. R., Kuehn, D. M., and Larson, H. K., "Investigation of separated flows in supersonic and subsonic streams with emphasis on the effect of transition," NACA Rept. 1356 (1958).
- <sup>3</sup> Kepler, C. E. and Bogdonoff, S. M., "Interaction of a turbulent boundary layer with a step at  $M = 3$ ," Princeton Univ., Dept. of Aeronautical Engineering, Rept. 238 (1953).
- <sup>4</sup> Mager, A., "On the model of a free shock-separated turbulent boundary layer," J. Aeronaut. Sci. **23**, 181-184 (1956).
- <sup>5</sup> Mager, A., "Prediction of shock-induced turbulent boundary layer separation," J. Aeronaut. Sci. **22**, 201-202 (1955).
- <sup>6</sup> Crocco, L. and Probst, R. F., "The peak pressure rise across an oblique shock emerging from a turbulent boundary layer over a plane surface," Princeton Univ., Dept. of Aeronautical Engineering, Rept. 254 (1954).
- <sup>7</sup> Burbank, P. D., Newlander, R. A., and Collins, I. K., "Heat transfer and pressure measurements on a flat plate surface and heat transfer measurements on attached protuberances in a supersonic turbulent boundary layer at Mach numbers of 2.65, 3.51, and 4.44," NASA TND-1372 (1962).

## Measurements of Panel Response to Turbulent Boundary-Layer Excitation

LUCIO MAESTRELLO\*

The Boeing Company, Renton, Wash.

### Nomenclature

$f$	= frequency in cycles per second
$M$	= Mach number
$\overline{p_w^2}$	= mean-square wall pressure fluctuation
$PWL$	= total acoustic power radiated
$R$	= correlation coefficient
$U$	= duct centerline velocity
$U_c$	= convection velocity
$\langle Y^2 \rangle$	= mean-square panel displacement
$\langle \bar{Y}^2 \rangle$	= total mean-square panel displacement
$\xi$	= incremental distance in direction of flow
$\tau$	= time delay
$\tau_w$	= wall shear stress

SOME preliminary results of an experimental investigation are presented concerning the motion induced by a turbulent boundary layer in rigidly supported panels and the ra-

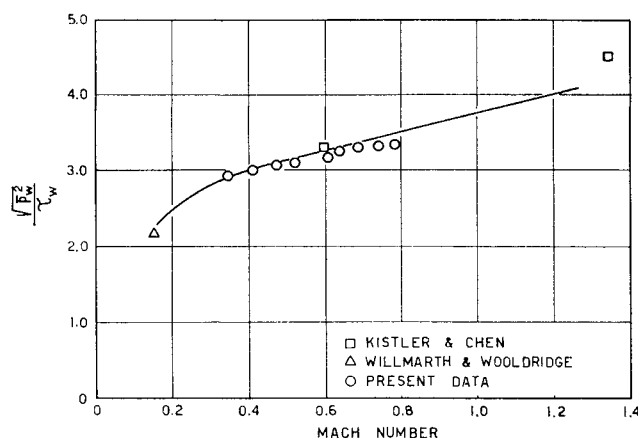


Fig. 1 Wall pressure fluctuations.

Received August 10, 1964.

\* Research Engineer, Research and Development Acoustics Group.

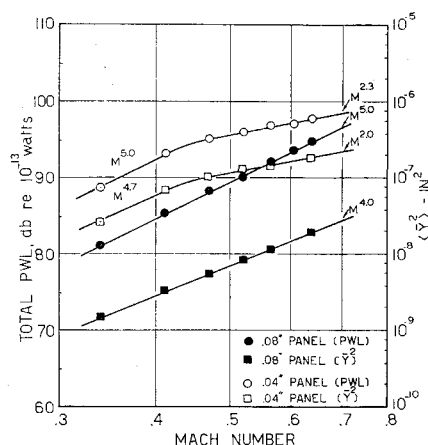


Fig. 2. Total acoustic power,  $PWL$  total mean square displacement,  $\langle \bar{Y}^2 \rangle$ .

diated acoustic field generated by such motion. The measurements were conducted at the Boeing Boundary Layer Test Facility consisting of 25 ft of duct with cross sectional area of approximately  $7.3 \times 3.5$  in. with velocities up to 700 fps. The thicknesses of the panels tested were 0.020, 0.040, 0.060 and 0.080 in. They were made by milling out an area of  $7 \times 12$  in. from  $\frac{3}{4}$ -in. aluminum plates to the required thickness.

### Forcing Field

A careful study of the pertinent parameters of the turbulent field in the duct was made with the following results: 1) the mean velocity distribution and the shear stress  $\tau_w$  obtained from the standard technique were found to be consistent with the published result of turbulent duct flows; 2) the measured mean square pressure fluctuation  $\overline{p_w^2}$  at the rigid wall of the duct are in excellent agreement with those presented by various investigators<sup>1, 2</sup> (Fig. 1); 3) the space-time correla-

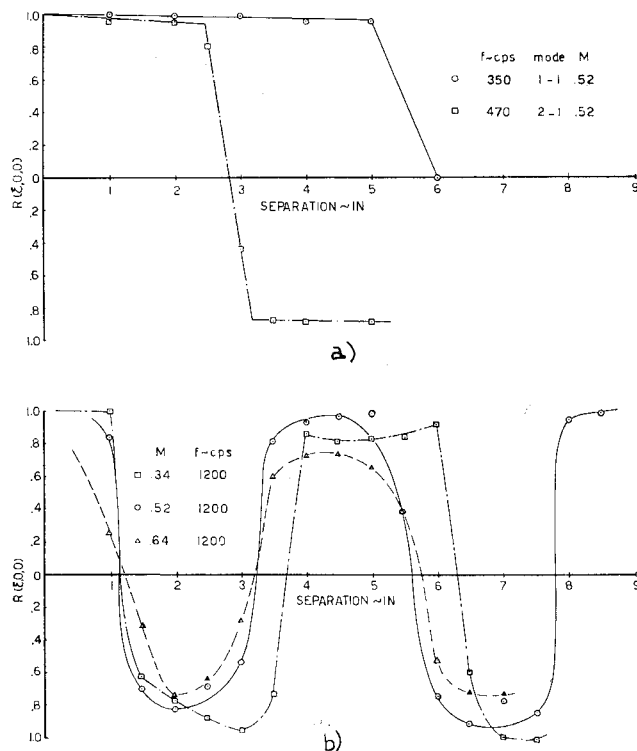


Fig. 3 Longitudinal space correlation of the displacement of a 0.040-in. panel: a) space correlation for Mach number 0.52 at various modal frequencies; b) space correlation for 1200 cps mode at various Mach numbers.

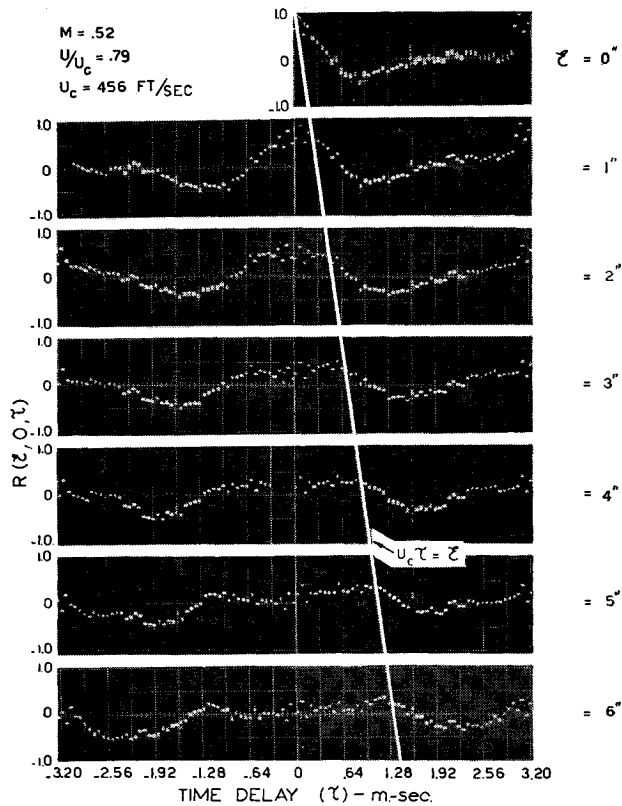


Fig. 4 Longitudinal space-time correlation of the panel displacement.

tion of the pressure fluctuations substantiates the well known results, that the convection velocity  $U_c$  of the pressure field is  $U_c = 0.79 U$  where  $U$  is the duct centerline velocity; and 4) the power spectrum distribution with the frequency has the same general form found by various authors.

#### Panel Motion

Subsequently, the response of the previously described four panels to these pressure fields has been investigated. The temporal mean square displacement of the panels along the longitudinal and lateral centerlines was measured. The spatially integrated values along these two lines are shown in Fig. 2 as a function of the Mach number. The motion of the 0.040-in. panel exhibits a substantially different behavior than that of the 0.080-in. panel.

A detailed study of various individual modes of excitation reveals the following interesting picture. In the lower modes, the thin panels vibrate in a standing wave pattern producing a square wave shaped space correlation (Fig. 3a). The panel motion in the higher modes and higher velocities changes character, however, as indicated by the sine wave shaped correlations (Fig. 3b), suggesting a moving wave pattern. Indeed the broadband longitudinal space-time correlation measurements of the displacement fluctuation (Fig. 4) clearly give evidence of a moving wave motion consistent with that of Ribner<sup>3</sup> who was the first to predict the existence of such running flexural waves. Also, the mean square displacement of the individual mode for the 0.040-in. panel (Fig. 5) indicates falling off in  $\langle Y^2 \rangle$  in the regions of high Mach number and mode number. The falling off is considered an effect of aerodynamic coincident. It is significant that the phase velocity coincides with the convection velocity of the turbulent pressure fluctuation in agreement with the observation of el Baroudi.<sup>4</sup> The correlation also indicates the existence of the moving surface waves having phase velocity of  $\pm U_c$ . A tentative interpretation of this diagram is as follows. Superimposed on standing waves, running flexural waves are propagating downstream with velocity  $U_c$ ; at the downstream edge

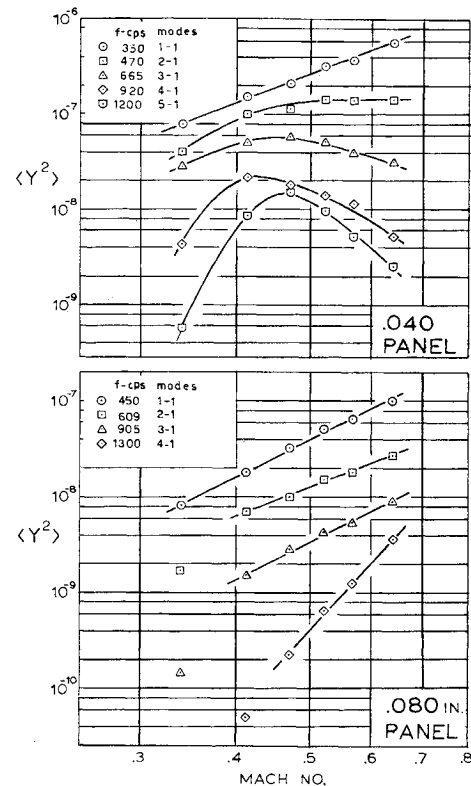


Fig. 5 Mean square modal displacement, in square inches

of the panel they reflect and propagate backward producing the correlation pattern seen in the region  $U_c \tau < 0$ .

Considering the motion of the 0.080-in.-thick panel, it is found that all the modes move essentially in a standing wave pattern. (The space correlation curves are square waves similar to those of Fig. 3a.) The correlation diagram of Fig. 6 shows no definite evidence of propagating waves. In addition, the total-mean-square displacement  $\langle \bar{Y}^2 \rangle$  varies as  $M^4$  throughout the velocity range in contrast to the thinner panels where the increase in amplitude is noticeably smaller at the higher speeds where the flexural waves appear (Fig. 2). A similar characteristic is observed in the change in mean-square displacement  $\langle Y^2 \rangle$  of individual modes (Fig. 5). The 0.080-in. panel displacement increases with Mach number, whereas the 0.040-in. panel displacement reaches a maximum at coincidence and then fall off with a further increase in Mach number.

#### Radiation Field

The difference in the motion of the thin and thick panels is reflected in the intensity of their radiation field and in their radiated power law (Fig. 2). The total acoustic power radiated

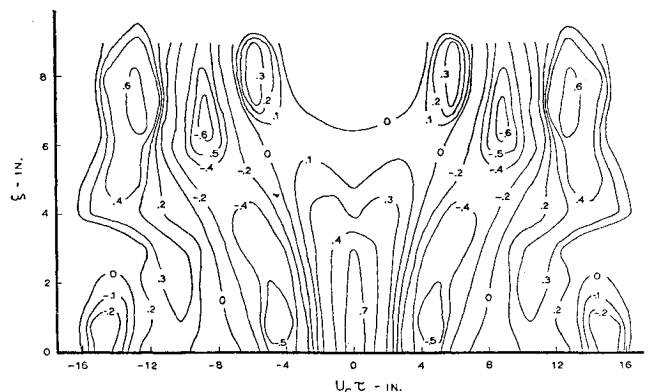


Fig. 6 Lines of constant longitudinal correlation of panel displacement.

(PWL) is proportional to  $M^5$  for standing or slowly moving wave patterns and  $M^{2.3}$  for the convected patterns with characteristic speeds equal to that of the turbulent eddies.

The effect of reflections on the radiation intensity is not well understood yet. Panels of various lengths are being investigated presently in order to clarify this question.

### References

- <sup>1</sup> Kistler, A. L. and Chen, W. S., "The fluctuating pressure field in a supersonic turbulent boundary layer," Jet Propulsion Lab. TR 32-277 (August 1962).
- <sup>2</sup> Willmarth, W. W. and Wooldridge, C. E., "Measurements of fluctuating pressure at the wall beneath thick turbulent boundary layer," University of Michigan Rept. 02970-1-T (April 1962).
- <sup>3</sup> Ribner, H. S., "Boundary-layer-induced noise in the interior of aircraft," University of Toronto Institute for Aerospace Studies Rept. 37 (April 1956).
- <sup>4</sup> el Baroudi, M. Y., "Turbulent-induced panel vibration," University of Toronto Institute of Aerospace Sciences Rept. 98 (February 1964).

## Nonequilibrium Electric Conductivity of Two-Phase Metal Vapors

A. W. ROWE\* AND J. L. KERREBROCK†

Massachusetts Institute of Technology, Cambridge, Mass.

**R**ANKINE cycle nuclear-electric power systems, with alkali metal working fluids and MHD generators used in place of turbines, show promise of producing considerably higher powers per unit of system mass than seem possible with turboalternators. A key problem in the development of such systems is the attainment of adequate electric conductivity in the alkali metals at the temperatures accessible to reactors.

Although it appears possible to attain high conductivities in seeded noble gases by utilizing the tendency for the electron temperature to be elevated by Joule heating, it has not been clear whether this effect would yield high conductivities in wet alkali-metal vapors.

The intrinsic difference is that formerly the free electrons and valence electrons formed an essentially closed thermodynamic system that is weakly coupled to the translational degrees of freedom of the gas, whereas in the present case the electron gas may be rather strongly coupled to the liquid phase through the processes of absorption and re-emission of the electrons by droplets of liquid.

An approximate theory of the wet nonequilibrium gas has been developed. Its principal features will be indicated.

The theory has been tested by comparison with experiments conducted in a high-temperature (up to 2000°K) potassium loop, whose principal features will also be indicated. More complete descriptions of both theory and experiment are given in Ref. 1.

### Theory

Consider a plasma in which all droplets are of the same size, but have varying charges  $Z$ . If the probability per unit

Received August 14, 1964. This research was supported in part by the U.S. Army, Navy, and Air Force under Contract DA36-039-AMC-03200(E), and in part by the U. S. Air Force (Aeronautical Systems Division) under Contract AF33(615)-1083 with the Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio.

\* Department of Mechanical Engineering and Research Laboratory of Electronics; formerly Research Officer, South African Atomic Energy Board.

† Associate Professor of Aeronautics and Astronautics and Research Lab. of Electronics. Member AIAA.

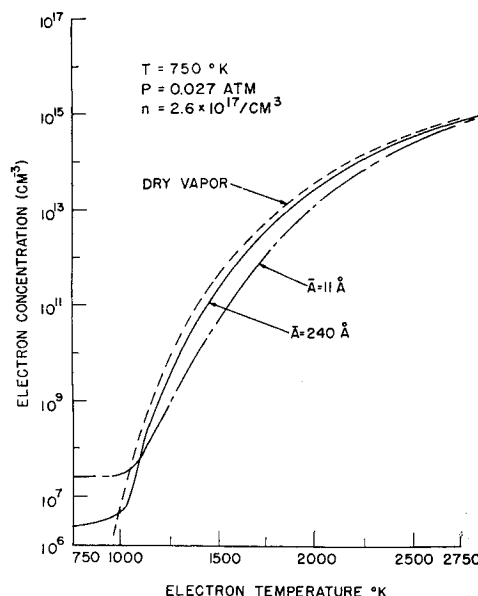


Fig. 1 Nonequilibrium ionization of potassium vapor as a function of electron temperature  $T_e$  for gas temperature  $T = 750^\circ\text{K}$ .

time that a drop will change from charge  $Z$  to  $Z - 1$  is denoted  $\alpha_{Z,Z-1}$ , the requirement for a steady population is as follows:

$$N_{Z-1}/N_Z = \alpha_{Z,Z-1}/\alpha_{Z-1,Z}$$

where  $N_Z$  is the number density of drops of charge  $Z$ .

A detailed development including electron and ion capture and thermionic emission, under the assumption of a Maxwellian electron and energy distribution, leads to the following expressions for the  $\alpha$ :

$$\left. \begin{aligned} \alpha_{Z,Z-1} &= N_e \left( \frac{kT_e}{2\pi m_e} \right)^{1/2} \left( 1 + \frac{W_Z - \phi_s}{kT_e} \right) \\ \alpha_{Z-1,Z} &= \frac{4\pi m_e}{h^3} (kT)^2 e^{-W_Z/kT} \left( 1 + \frac{W_Z - \phi_s}{kT} \right) + N_i \left( \frac{kT}{2\pi m_a} \right)^{1/2} e^{-V_i/kT} \end{aligned} \right\} Z \geq 1$$

$$\left. \begin{aligned} \alpha_{Z,Z-1} &= N_e \left( \frac{kT_e}{2\pi m_e} \right)^{1/2} e^{(W_Z - \phi_s)/kT_e} \\ \alpha_{Z-1,Z} &= \frac{4\pi m_e}{h^3} (kT)^2 e^{-\phi_s/kT} + N_i \left( \frac{kT}{2\pi m_a} \right)^{1/2} \left( 1 - \frac{V_i}{kT} \right) \end{aligned} \right\} Z \leq 0$$

where

$$W_Z = \phi_s + \left[ \frac{3}{8} + Z - 1 \right] (e^2/4\pi\epsilon_0 A)$$

$$V_i = (Z - \frac{3}{8}) e^2/4\pi\epsilon_0 A$$

Here,  $A$  is the droplet radius,  $\phi_s$  is the work function of the flat liquid surface, and the rest of the notation is conventional.

The resultant expressions for  $N_{Z-1}/N_Z$  plus Saha's equation for the atomic ionization, together with mass and charge conservation conditions, have been used in a computer program to study the ionization of wet potassium vapor as a function of electron temperature. Figure 1 indicates the essential results. Although the equilibrium ionization depends on the droplet size, over a considerable range ( $A$  greater than 50 Å), it is well approximated by

$$N_e = 2 \left( \frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\phi_s/kT}$$